Tentamen Mechanica dd 19-4-2010

Opgave 1

a) $m\ddot{x} = -\gamma x$

b) $\ddot{x} + \frac{\gamma}{m}\dot{x} = 0 \rightarrow \dot{x}(t) = A e^{-\frac{\gamma}{m}t}$

$\dot{x}(0) = A e^{0} = A$
$\dot{x}(0) = v_0$
} $A = v_0$

} $\dot{x}(t) = v_0 e^{-\frac{\gamma}{m}t}$

c) $x(t) = \int_0^t \dot{x}\, dt + x_0$

$= x_0 + v_0 \int_0^t e^{-\frac{\gamma}{m}t} dt = x_0 - \frac{v_0 m}{\gamma}\left[e^{-\frac{\gamma}{m}t}\right]_0^t$

$= x_0 - \frac{v_0 m}{\gamma}\left[e^{-\frac{\gamma}{m}t} - 1\right] = x_0 + \frac{v_0 m}{\gamma}\left(1 - e^{-\frac{\gamma}{m}t}\right)$

Opgave 2

a) Wet van behoud van mechanische energie

b) $mgh = \frac{1}{2} m v_B^2 \rightarrow v_B = \sqrt{2gh}$

c) $mgh = \frac{1}{2} m v_D^2 + mg2a \rightarrow v_D = \sqrt{2g(h-2a)}$

d) centrifugaalkracht

e) $F_{cf} = m\omega^2 a = \frac{mv^2}{a}$ (omhoog bij D)

$F_z + F_{normaal} = F_{cf}$

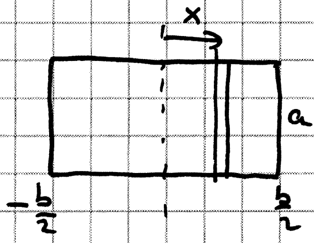
kogel komt los als $F_{normaal} = 0 \rightarrow$ als $F_{cf} = F_z$

$\therefore \frac{mv^2}{a} = mg \rightarrow \frac{v^2}{a} = g \rightarrow 2g(h-2a) = ga$

$2(h-2a) = a$

$\rightarrow h = \frac{5}{2}a$

Opgave 3

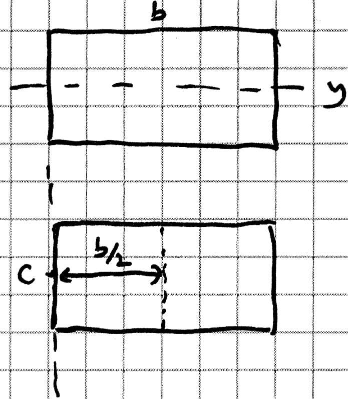


a) $I_y = \int x^2\, dm$ $\qquad dm = a\, dx\, \sigma$

$$= \int_{-\frac{b}{2}}^{\frac{b}{2}} a\sigma x^2\, dx = a\sigma \left[\frac{1}{3}x^3\right]_{-\frac{b}{2}}^{\frac{b}{2}} =$$

$$= \frac{a\sigma}{3}\left(\frac{b^3}{8} - \left(-\frac{b^3}{8}\right)\right) = \frac{1}{12}ab^3\sigma = \frac{1}{12}Mb^2$$

b) Zoooh $\quad I_x = \int y^2\, dm = \frac{1}{12}Ma^2$



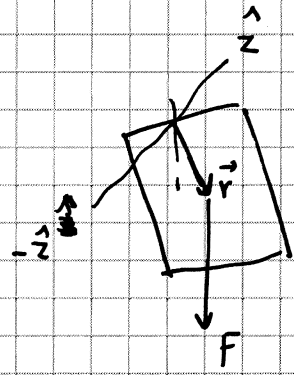
$$I_{yc} = I_{ycm} + m\left(\frac{b}{2}\right)^2 \quad \text{(parallelle assen)}$$

$$= \frac{1}{12}Mb^2 + \frac{1}{4}Mb^2 = \frac{1}{3}Mb^2$$

$$I_z = I_x + I_y = \frac{1}{12}Ma^2 + \frac{1}{3}Mb^2$$

(loodrechte assen!)

c) $\vec{N} = \frac{d\vec{L}}{dt} \longrightarrow \vec{r} \times \vec{F} = I\frac{d\vec{\omega}}{dt}$



$$+\frac{b}{2}Mg\sin\theta\,\hat{z} = -I\ddot{\theta}\,\hat{z}$$

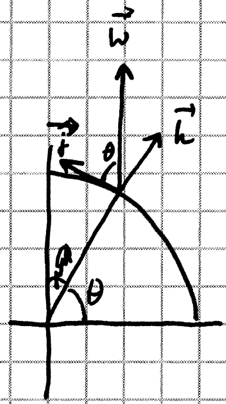
$$\therefore I\ddot{\theta} + \frac{b}{2}Mg\sin\theta = 0$$

klein θ $\qquad I\ddot{\theta} + \frac{b}{2}Mg\,\theta \simeq 0$

$$\ddot{\theta} + \frac{bMg}{I}\theta = 0 \rightarrow \omega = \sqrt{\frac{bMg}{I}} \rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{bMg}}$$

$$T = 2\pi\sqrt{\frac{\frac{1}{12}Ma^2 + \frac{1}{3}Mb^2}{bMg}} = 2\pi\sqrt{\frac{\frac{a^2}{12} + \frac{b^2}{3}}{bg}} = 2\pi\sqrt{\frac{a^2}{12bg} + \frac{b}{3g}}$$

Opgave 4



a) $$m\ddot{\vec r} = -mg\vec k - \gamma|\vec v|\vec v - 2m\vec\omega\times\vec v$$

b) $$\vec\omega = \omega\cos\theta\,\vec j + \omega\sin\theta\,\vec k$$

zie figuur

c) $$m\ddot x = -\gamma\sqrt{\dot x^2+\dot y^2+\dot z^2}\,\dot x - 2m(\vec\omega\times\vec v)_x$$
$$m\ddot y = -\gamma\sqrt{\dot x^2+\dot y^2+\dot z^2}\,\dot y - 2m(\vec\omega\times\vec v)_y$$
$$m\ddot z = -mg - \gamma\sqrt{\dot x^2+\dot y^2+\dot z^2}\,\dot z - 2m(\vec\omega\times\vec v)_z$$

$$\vec\omega\times\vec v = \begin{pmatrix}\omega_y\dot z - \omega_z\dot y\\ \omega_z\dot x - \omega_x\dot z\\ \omega_x\dot y - \omega_y\dot x\end{pmatrix} = \begin{pmatrix}\omega\cos\theta\,\dot z - \omega\sin\theta\,\dot y\\ \omega\sin\theta\,\dot z\\ -\omega\cos\theta\,\dot x\end{pmatrix}$$

$$\therefore\ m\ddot x = -\gamma\sqrt{\dot x^2+\dot y^2+\dot z^2}\,\dot x - 2m\omega\cos\theta\,\dot z + 2m\omega\sin\theta\,\dot y$$
$$m\ddot y = -\gamma\sqrt{\dot x^2+\dot y^2+\dot z^2}\,\dot y - 2m\omega\sin\theta\,\dot z$$
$$m\ddot z = -mg - \gamma\sqrt{\dot x^2+\dot y^2+\dot z^2}\,\dot z + 2m\omega\cos\theta\,\dot x$$

d) niet separabel

De x-component v.d. vergelijking bevat ook $\dot y$ en $\dot z$ etc.